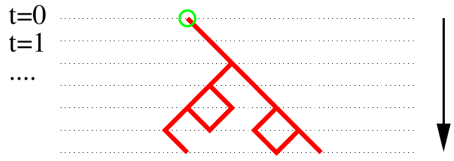

Directed bond percolation

Exercise 12.1: Bond-DP evaluated exactly (6P)

In this exercise we consider a directed bond percolation process on an infinite chain. The model is controlled by the probability p that a bond is open.

(a) Suppose that the process starts with a single active site in the center at $t = 0$ while all other sites are inactive. Compute the complete probability distribution for the four configurations at $t = 1$ and the eight possible configurations at $t = 2$. (1P)

(b) Compute the survival probability (i.e., the probability to find at least one active particle) for $t = 1, 2$ as a function of p. (2P)

(c) Suppose that the process starts with a *fully occupied* lattice (all sites active) at $t = 0$. Compute the density of active sites (the expectation value that a site is active) at $t = 1, 2$ as a function of p. (2P)

(d) Compare the two quantities and explain you observation. (1P)

Exercise 12.2: DP mean field approximation (6P)

In the mean field approximation of directed percolation (DP) extended by a term for diffusion, the density of active sites $\rho(\mathbf{x}, t)$ evolves according to the partial differential equation

$$\dot{\rho} = \lambda\rho(1 - \rho) - \rho + D\nabla^2\rho.$$

(a) Determine the homogeneous (space-independent) stationary (time-independent) density ρ_{stat} as a function of λ. Where is the critical point λ_c? Test the solutions for stability by a stability analysis. (2P)

(b) Find the exponent for $\rho_{stat} \sim (\lambda - \lambda_c)^\beta$ near the critical point. (1P)

(c) At criticality $\lambda = \lambda_c$ the homogeneous density $\rho(t)$ decays as $\rho(t) \sim t^{-\delta}$. Compute the exponent δ. (1P)

(d) Show that the Greens function of the mean field equation at the critical point be written in the form

$$G(\mathbf{x}, t) = t^{-1} f(x^2/t).$$

Hint: It is sufficient to derive an autonomous differential equation for f, you don't have to compute f explicitly. Restrict yourself to $t > 0$ and ignore the scaling of the δ-functions. (2P)

(Σ = 12P)

To be submitted electronically on Wednesday, July 20, 2022, via WueCampus according to our guidelines on the web page `cs.hayehinrichsen.de`.