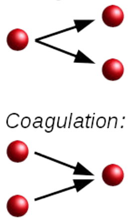
B. Derrida, one of the pioneers of matrix product states.

EXERCISE 5.1: COAGULATION-DECOAGULATION PRODUCT STATE (3P)

Consider the coagulation-decoagulation process with

- hopping to the left $0A \to A0$ at rate w_L,
- hopping to the right $A0 \to 0A$ at rate w_R,
- coagulation $AA \to 0A$ and $AA \to A0$ with rate w_C
- decoagulation $0A \to AA$ and $A0 \to AA$ with rate w_D.



on 1D chain with L sites. All four rates are assumed to be positive.

(a) Set up the two-site Liouville operator $\mathcal{L}^{(2)}$. (1P)

(b) How do we have to tune the rates so that the stationary state is a nontrivial homogeneous product state (nontrivial means that it is neither the fully occupied nor the empty lattice)? How large is the average density of particles in this case? (2P)

EXERCISE 5.2: MATRIX PRODUCT STATES FOR THE ASEP (9P)

Let us consider the ASEP with particle injection and removal at the boundaries, choosing the hopping rates $w_R = \lambda$ und $w_L = \lambda^{-1}$. In the lecture we have shown that there is a special line parametrized by $\alpha + \beta = \lambda - \lambda^{-1}$, where the matrix product state reduces to an ordinary product state without any correlations. Moreover, it was shown that in general the matrix representations for the ASEP are infinite-dimensional. In this exercise we show that there is another line in the parameter space along which a two-dimensional representation exists. For solving this exercise use *Mathematica*® or similar software.

(a) Prove that for $(\alpha + \lambda^{-1})(\beta + \lambda^{-1}) = 1$ the two-dimensional matrices and vectors

$$\tilde{\mathbf{E}} = \begin{pmatrix} \frac{1}{\alpha} & 0 \\ 1 & \frac{1+\alpha\lambda}{\alpha\lambda^2} \end{pmatrix}, \quad \tilde{\mathbf{D}} = \begin{pmatrix} \frac{1}{\beta} & -1 \\ 0 & \frac{1+\beta\lambda}{\beta\lambda^2} \end{pmatrix}, \quad \langle\tilde{\alpha}| = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad |\tilde{\beta}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

are a representation of the ASEP algebra (3P)

$$\lambda\tilde{\mathbf{D}}\tilde{\mathbf{E}} - \lambda^{-1}\tilde{\mathbf{E}}\tilde{\mathbf{D}} = \tilde{\mathbf{D}} + \tilde{\mathbf{E}}, \quad \langle\tilde{\alpha}|\tilde{\mathbf{E}} = \alpha^{-1}\langle\tilde{\alpha}|, \quad \tilde{\mathbf{D}}|\tilde{\beta}\rangle = \beta^{-1}|\tilde{\beta}\rangle .$$

Please turn over ⇒

(b) Choose λ according to the condition in (a) and compute the four-component vector

$$|P\rangle = \frac{1}{\mathcal{N}} \Big(\langle\tilde{\alpha}|\tilde{\mathbf{E}}\tilde{\mathbf{E}}|\tilde{\beta}\rangle\,, \quad \langle\tilde{\alpha}|\tilde{\mathbf{E}}\tilde{\mathbf{D}}|\tilde{\beta}\rangle\,, \quad \langle\tilde{\alpha}|\tilde{\mathbf{D}}\tilde{\mathbf{E}}|\tilde{\beta}\rangle\,, \quad \langle\tilde{\alpha}|\tilde{\mathbf{D}}\tilde{\mathbf{D}}|\tilde{\beta}\rangle \Big)$$

of a chain with two sites, where

$$\mathcal{N} = \langle\tilde{\alpha}|\tilde{\mathbf{C}}\tilde{\mathbf{C}}|\tilde{\beta}\rangle\,, \quad \tilde{\mathbf{C}} = \tilde{\mathbf{D}} + \tilde{\mathbf{E}}\,,$$

is the normalization. Apply the full 4×4 Liouvillian on a chain with two sites directly to $|P\rangle$ in order to verify that this vector is indeed the stationary state. (3P)

(c) Compute the bare correlation function C_{12} and its connected part C_{12}^{conn} in the stationary state on a chain with two sites. (3P)

(Σ = 12P)

To be submitted electronically on Wednesday, June 01, 2022, via WueCampus according to our guidelines on the web page `cs.hayehinrichsen.de`.