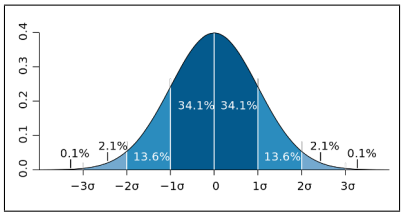
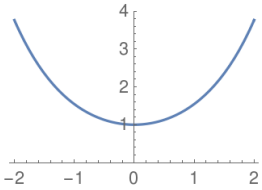
# Physics of Complex Systems

Lecture and Tutorials – Prof. Dr. Haye Hinrichsen – B. Sc. Thomas Siedler – SS 2022




Normal distribution [Wikimedia]

## Exercise 2.1: Transforming probability densities (6P)




Consider the curve $y = \cosh x$. The aim of this exercise is to decorate this curve in the interval $x \in [x_1, x_2]$ with random points in such a way that the density of the points is uniform along the curve. Since the slope of the curve varies, this means that the $x$-coordinates of these points are *not* uniformly distributed.

(a) Compute the probability density $p(x)$ of the $x$-coordinates. Hint: the density of the points per arc length of the curve has to be constant. (2P)

(b) Normalize the probability density on the interval $x \in [x_1, x_2]$. (1P)

(c) Find a function $f : z \mapsto x = f(z)$ such that it maps a uniform probability density $p(z) = const$ to the non-uniform probability density $p(x)$ calculated in (a)-(b). (2P)

(d) Adjust the integration constant in (c) in such a way that $f$ maps $[z_1, z_2] \mapsto [x_1, x_2]$ with $f(z_1) = x_1$ and $f(z_2) = x_2$. Specialize the result for a standard random number ($z_1 = 0, z_2 = 1$). (1P)

## Exercise 2.2: Normal distribution (6P)

The probability density function of the normal distribution with zero mean and variance $\sigma^2$ is given by

$$p(x) \;=\; \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}} \,.$$

(a) Derive a recursion relation for the moments which expresses $m_n$ in terms of $m_{n-2}$. The relation can be derived by partial integration of the defining integral. (2P)

(b) Apply this recursion relation to compute the first six moments $m_1, \ldots, m_6$. (1P)

(c) Derive the moment-generating function of the normal distribution given above. Hint: Try quadratic completion (quadratische Ergänzung) in the integrand. (2P)

(d) Compute all cumulants $\kappa_n$ of the normal distribution given above from the cumulant-generating function. (1P)

**($\Sigma$ = 12P)**

To be submitted electronically on Wednesday, May 11, 2022, via WueCampus according to our guidelines on the web page `cs.hayehinrichsen.de`.