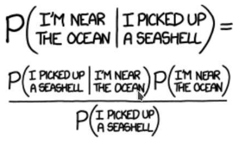
Physics of Complex Systems

Lecture and Tutorials – Prof. Dr. Haye Hinrichsen – B. Sc. Thomas Siedler – SS 2022



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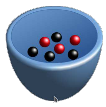
Exercise 1.1: Bayes Theorem (2P)

An urn contains three red and four black balls. Two balls are randomly drawn from the urn (without putting them back). What is the probability that the first one is black given that the second one is red?



Exercise 1.2: Poisson distribution (6P)

The poisson distribution $P_\lambda(k) = \frac{\lambda^k}{k!}e^{-\lambda}$ can be understood as the limit of the binomial distribution in the case of "rare events".

(a) Let $p = \lambda/N$ and take $N \to \infty$ while keeping λ and k constant. Show that in this limit we can approximate $(1-p)^{N-k} \approx e^{-\lambda}$. (1P)

(b) Show similarly that $\binom{N}{k} \approx \frac{N^k}{k!}$. (1P)

(c) Use (a) and (b) to show that in this limit the binomial distribution tends to the Poisson distribution. (1P)

(d) Check that the Poisson distribution is properly normalized. (1P)

(e) Compute the moment- and cumulant-generating functions. (1P)

(f) Determine all cumulants. (1P)

Exercise 1.3: Reconstruction of a probability density (4P)

(a) Prove the following statement: If the moment-generating function $M_X(t)$ is analytic, then the corresponding probability density $p(x)$ is given by the inverse Fourier transform (1P)

$$p(x) \;=\; \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}s\, e^{-ixs} M_X(is).$$

(b) Consider a probability distribution with the cumulants

$$\{\frac{\kappa_0}{0!}, \frac{\kappa_1}{1!}, \frac{\kappa_2}{2}, \frac{\kappa_3}{3!}, \ldots\} \;=\; \{0, 0, \frac{3}{2}, 0, -\frac{1}{2}, 0, \frac{1}{3}, 0, -\frac{1}{4}, 0, \frac{1}{5}, 0, -\frac{1}{6}, \ldots\}$$

Compute the generating functions $K(t)$ and $M(t)$. (2P)

(c) Use (a) to reconstruct the probability density $p(x)$. (1P)

(Σ = 12P)

To be submitted electronically on Wednesday, May 04, 2022, via WueCampus according to our guidelines on the web page `cs.hayehinrichsen.de`.