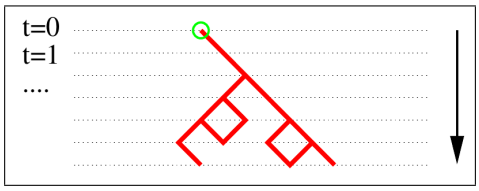


Bond directed percolation

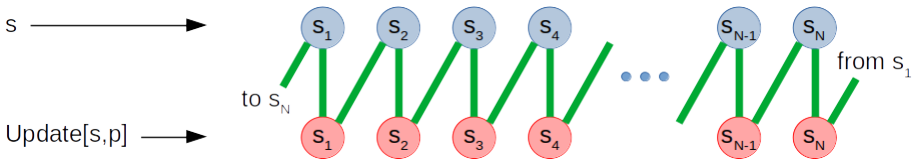
EXERCISE 9.1: NUMERICAL SIMULATION OF DIRECTED PERCOLATION (6P)

The aim of this exercise is to simulate a directed bond percolation process numerically. This can be done in *Mathematica*® (and we will use a Mathematica-like syntax here) but you are free to use any other programming language as well (such as Python or C++). However, you are requested to keep the code compact and understandable so that it can be corrected easily.

(a) Let S be a list of N elements $S = \{s_1, s_2, \ldots, s_N\}$ representing the state of a horizontal row in a directed bond percolation process, where

$$s_i = \begin{cases} 1 & \text{if the site is occupied/active/wet/infected} \\ 0 & \text{if the site is empty/inactive/dry/healthy} \end{cases}$$

Write a function `update[S,p]` that returns the updated row according to the following update scheme:



Here the green lines stand for bonds that are open with probability p and closed otherwise (compared to the lecture notes, the bonds are tilted to the left in order to realize the update on a single list). Note that the leftmost and the rightmost site are connected by a green line, implementing periodic boundary conditions. (2P)

(b) Write another function that returns the density of active sites of the list S. (1P)

(c) Perform a single run of the simulation at the critical point: Start the simulation with a fully occupied lattice $S = \{1, 1, 1, \ldots, 1\}$ with $N = 1000$ sites. Simulate the process for $p = 0.6447$ over $T = 1000$ time steps and plot the density double-logarithmically as a function of time (for example using `ListLogLogPlot`). (1P)

(d) Compute the mean value of the density as a function of time averaged over $M = 100$ stochastically independent runs (this can take several minutes) and plot the average double-logarithmically as a function of time. (1P)

(e) Fit the function $\rho(t) = At^{-B}$ to the average data obtained in (d) estimating the constants A, B. Hint: In *Mathematica*® you can simply use the tool `FindFit`. (1P)

Please turn over ⇒

Exercise 9.2: DP mean field approximation (6P)

In the mean field approximation of directed percolation (DP) extended by a term for diffusion, the density of active sites $\rho(\mathbf{x}, t)$ evolves according to the partial differential equation

$$\dot{\rho} = \lambda\rho(1-\rho) - \rho + D\nabla^2\rho.$$

(a) Determine the homogeneous (space-independent) stationary (time-independent) density ρ_{stat} as a function of λ. Where is the critical point λ_c? Test the solutions for stability by a stability analysis. (2P)

(b) Find the exponent for $\rho_{stat} \sim (\lambda - \lambda_c)^\beta$ near the critical point. (1P)

(c) At criticality $\lambda = \lambda_c$ the homogeneous density $\rho(t)$ decays as $\rho(t) \sim t^{-\delta}$. Compute the exponent δ. (1P)

(d) Show that the Greens function of the mean field equation at the critical point be written in the form

$$G(\mathbf{x}, t) = t^{-1} f(x^2/t).$$

Hint: It is sufficient to derive an autonomous differential equation for f, you don't have to compute f explicitly. Restrict yourself to $t > 0$ and ignore the scaling of the δ-functions. (2P)

(Σ = 12P)

To be handed in electronically until Wednesday, June 24, 2020, 12:00, on WueCampus according to our Corona guidelines on the web page `cs.hayehinrichsen.de`.