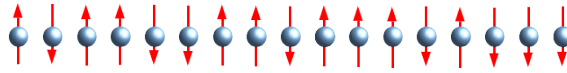


PHYSICS OF COMPLEX SYSTEMS

LECTURE AND TUTORIALS – PROF. DR. HAYE HINRICHSEN – B. SC. NILS PLÄHN – SS 2020



EXERCISE 8.1: 1D ISING MODEL WITH FREE BOUNDARY CONDITIONS (10P)

Let us consider the one-dimensional Ising model of N classical spins $s_n = \pm 1$ with free (non-periodic) boundary conditions

$$E_N = - \sum_{n=1}^{N-1} s_n s_{n+1}.$$

- Compute the partition sum $Z_N = \sum_{\{s\}} e^{-\beta E_N}$ recursively. (2P)
- Compute the free energy F , the entropy H , the internal energy U , and the heat capacity $C = T \frac{\partial H}{\partial T}$ (you may set $k_B = 1$). (2P)
- Prove that $\langle s_j \rangle = 0$. (1P)
- Show that the connected spin correlation function $G_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$ decays exponentially with increasing distance $r = |j - i|$ as $G_{ij} \sim e^{-r/\xi}$. Show that the correlation length is given by $\xi = -\ln(\tanh \beta)^{-1}$. Interpret the result in the limit $T \rightarrow 0$. *Hint:* $\langle s_i s_j \rangle = \langle s_i s_{i+1} s_{i+1} s_{i+2} s_{i+2} \dots s_{j-1} s_{j-1} s_j \rangle$ (3P)
- Calculate the magnetic susceptibility $\chi(\beta)$ in zero magnetic field $h = 0$ by using the so-called fluctuation-dissipation theorem

$$\frac{\chi(\beta)}{N} = \beta \sum_{n=1}^N G_{\frac{N}{2}n}$$

in the thermodynamic limit $N \rightarrow \infty$. For simplicity assume N to be even. What happens for $T \rightarrow 0$? (2P)

EXERCISE 8.2: BOGOLIUBOV TRANSFORMATION (2P)

A Bogoliubov transformation is a canonical transformation that mixes creation and annihilation operators without destroying the commutation relations.

Let ψ_1, ψ_1^\dagger and ψ_2, ψ_2^\dagger be two pairs of fermionic operators, that is $\{\psi_i^\dagger, \psi_j^\dagger\} = \{\psi_i, \psi_j\} = 0$ and $\{\psi_i, \psi_j^\dagger\} = \delta_{ij}$. Find coefficients $u, v \in \mathbb{C}$ such that

$$\eta = u\psi_1 + v\psi_2^\dagger, \quad \eta^\dagger = u^*\psi_1^\dagger + v^*\psi_2$$

obey fermionic anticommutation relations and find a general parametrization of u and v .

($\Sigma = 12P$)

To be handed in electronically until Wednesday, June 17, 2020, 12:00, on WueCampus according to our Corona guidelines on the web page cs.hayehinrichsen.de.