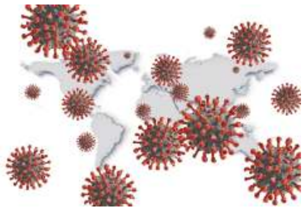

Warmup Exercise

Exercise 1.1: SIR model applied to the Corona pandemic (6P)

Consider the SIR-model (see lecture notes)

$$\begin{aligned}\dot{S}(t) &= -\beta I(t)S(t)\\ \dot{I}(t) &= +\beta I(t)S(t) - \gamma I(t)\\ \dot{R}(t) &= +\gamma I(t).\end{aligned}$$

with the initial conditions $S(0) = 1 - \epsilon$, $I(0) = \epsilon$, and $R(0) = 0$ where $\epsilon \ll 1$. For solving this exercise, please install the student version of *Mathematica*® from the Rechenzentrum or use another software tool of your choice.

(a) Show that the number of infected individuals increases exponentially at the beginning of the outbreak. (1P)

(b) Derive a relation between $I(t)$ and $S(t)$. (2P)
Hint: Consider the quotient $\dot{I}(t)/\dot{S}(t)$, separate variables, and integrate the resulting differential equation.

(c) Use (b) to determine the peak value of the infections I_{max}. How does the peak value depend on β? Convince yourself that *social distancing* lowers I_{max}. (1P)

(d) Solve the system of differential equations numerically with *Mathematica*® (`NDSolve`) or a similar software tool using the parameters $\beta = 0.3$, $\gamma = 0.05$, and $\epsilon = 0.01$. Plot the three quantities in a single graph as a function of time in the range $t = 0 \ldots 60$ and verify your result in (c). (2P)

(Σ = 6P)

This warmup exercise has only half as many points as usual. To be handed in electronically on Wednesday, April 29, 2020, according to our Corona guidelines on the web page `cs.hayehinrichsen.de`.